Int. J. Heat Mass Transfer. Vol. 7, pp. 955–968. Pergamon Press 1964. Printed in Great Britain

EXPERIMENTAL STUDY OF LAMINAR CONVECTION IN THE CHANNEL BETWEEN PARALLEL PLATES WITH UNIFORM HEAT SOURCES IN THE FLUID

J. L. NOVOTNY* and E. R. G. ECKERT
Heat Transfer Laboratory, University of Minnesota, U.S.A.

(*Received* 11 *December* 1963 *and in revised form* 20 *March* 1964)

Abstract—Heat-transfer results for the free convective flow of a heat-generating fluid in a vertical parallel-plate channel have been obtained through use of a Zehnder–Mach interferometer. The experiments were carried out with a parallel-plate channel, open at the top and bottom, suspended in a large tank of dilute electrolyte. The heat-transfer characteristics of the channel were studied starting from an initial state of uniform temperature in the whole system (no flow) to the onset of quasi-steady convection when a step change in heat generation is applied to the fluid initially between the walls of the channel.

Experimental results are presented for channel spacings of 0·197 and 0·745 in and for an internal heat generation rate of approximately 13 000 Btu/h ft^3. These results are compared with theoretical analyses for the early transient period and for the final quasi-steady state. The agreement between experiment and theory indicates that the heat transfer for the range of parameters studied can be predicted adequately for the whole transient period.

NOMENCLATURE

a, half-width of duct;
A, constant, see equation (1);
B, constant, see equation (5);
c, specific heat;
k, thermal conductivity;
q, heat flux rate at wall;
$\dot{q}$, heat generation rate;
Re, average Reynolds number, $4\bar{u}a/\nu$;
t, time from estimated start of heating;
T, temperature;
u, velocity;
$\bar{u}$, average velocity in channel;
x, co-ordinate (Fig. 2);
y, co-ordinate (Fig. 2);
ν, kinematic viscosity;
ρ, density.

Subscripts
c, centerline;
0, upstream of duct entrance;
w, wall;
$(\ )_0$, entrance value.

* Present address: Department of Mechanical Engineering, University of Delaware, U.S.A.

INTRODUCTION

THE STUDY of heat transfer for flow in tubes with heat sources uniformly distributed in the fluid has been stimulated in recent years by applications in nuclear reactors and the chemical industries. However, previous investigations have concentrated only on two fully developed flow situations; namely, combined free and forced flow and pure forced flow.

Hallman [1] studied combined laminar flow in vertical circular tubes under steady fully-developed hydrodynamic and thermal conditions. Analytical results were obtained for the boundary conditions of constant wall temperature and constant heat flux. For the analogous problem in a parallel-plate channel, the analytical treatment was carried out by Ostrach [2, 3]. Reference 2 presents results for a spatially constant wall temperature when the flow through the channel is caused solely by buoyancy forces, and reference 3 studies a duct with linearly varying wall temperature where a forced flow is superimposed on the system. Poppendick and Palmer have considered laminar fully developed forced convection in a circular tube

[4, 5] and in a parallel-plate channel [6]. Measurements of the fully developed wall-to-bulk-temperature difference are given in [5] for a thermally insulated wall. The Grätz problem in circular tubes, which covers the thermal entrance region in steady fully developed laminar forced flow, has been extended by Topper [7] to include internal heat generation with a spatially constant wall temperature and by Sparrow and Siegel [8] for constant wall heat flux. Sparrow, Novotny, and Lin [9] considered the same problem for a parallel-plate channel with the boundary conditions of constant wall temperature and constant heat flux. Recently, Inman [10] reported experimental wall-to-bulk-temperature differences under boundary conditions which match those of the analysis given in [8]. In contrast to the rather complete investigation of the fully developed region for the flow of fluids with internal heat sources, no experimental or theoretical studies have been reported for the region of simultaneously developing flow and temperature fields.

The present paper describes an experimental investigation of the laminar flow of a fluid with uniform internal heat generation in a vertical parallel-plate channel. It presents results for the situation where the channel is suspended in an infinite fluid and where the flow is caused solely by buoyancy forces set up by the thermal conditions imposed on the system. Information on the temperature field in the fluid between the channel walls was obtained by use of a Zehnder–Mach interferometer. The study includes the range of time from an initial state of uniform temperature in the whole system (no flow) to a quasi-steady state when a step change in heat generation is applied to the fluid initially between the walls of the channel.

DESCRIPTION OF PROBLEM

A more detailed discussion of the problem considered in the present investigation is appropriate at this point. A vertical parallel-plate channel open at the top and bottom is immersed in an infinite fluid. Initially, the temperature of the channel walls is uniform and equal to that of the fluid which is at rest. At time equal zero, a spatially and timewise constant internal heat generation is suddenly imposed on the fluid in the space between the channel walls. The simplest boundary condition would be to maintain the wall temperature at its original value. This, however, is very difficult to obtain experimentally and the wall temperature was, therefore, allowed to vary with time. The variation during the transient period was approximately linear, and due to the heat capacity of the walls it lagged continuously behind the fluid temperature in the channel.

During the transient period the temperature field is divided into two regions. The first region contains the fluid which was originally in the channel. This fluid is heated through the internal heat generation and loses heat by conduction to the channel walls. Each fluid particle in this region is subjected to this process for the same length of time. Therefore, the temperature at any instant varies only in a direction normal to the channel walls, and the temperature field in the fluid is described by the unsteady one-dimensional heat conduction equation. This is the case regardless of the fact that the fluid in the channel begins to rise immediately on heating. Deviations from this occur near the exit of the duct due to end effects. The second region consists of the fluid particles which were originally outside of the channel. These fluid particles are subjected to the heating process for different lengths of time depending on when they entered the channel and where they are located relative to the walls. Correspondingly, the temperature field in this region also changes in the axial direction. The boundary between both regions moves in an upward direction with time, and steady or, for the present wall boundary conditions, quasi-steady state is established when this boundary passes out of the upper opening of the duct. The boundary between the two regions diffuses in time because of longitudinal conduction. This, however, is a comparatively slow process for fluids with normal Prandtl numbers.

The physical phenomena described above are exhibited in Fig. 1 which shows four enlargements of interferograms taken for plate spacings of 0·745 and 0·197 in. For each spacing, two interferograms are presented: one for a value of time when the entrance effects are still propagating through the channel, Figs. 1(a) and 1(c); the other for a value of time after the entrance

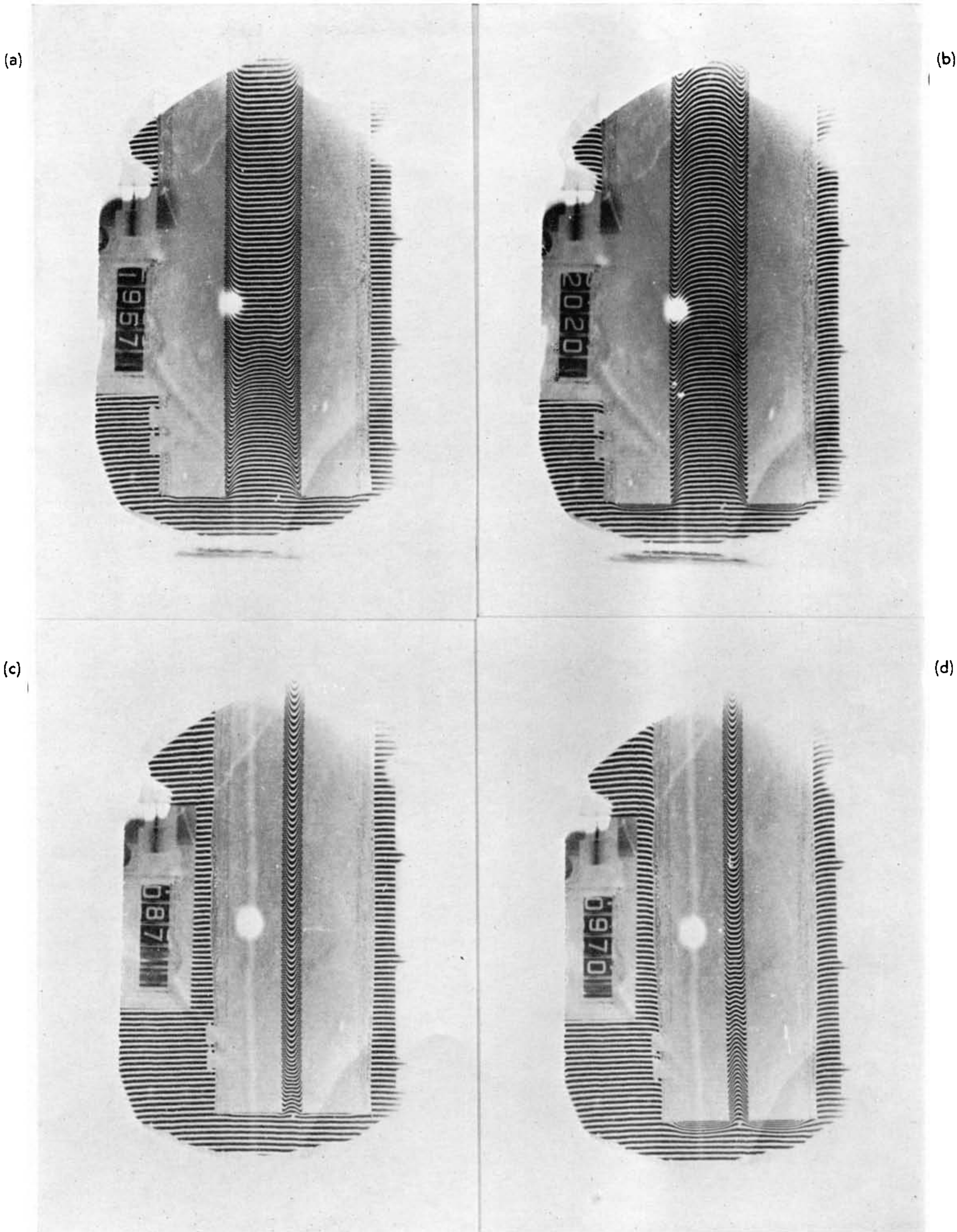

FIG. 1. Interferograms illustrating the temperature field in a parallel-plate channel with internal heat generation in the fluid for unsteady (a, c) and quasi-steady (b, d) conditions.

[*facing p.* 956]

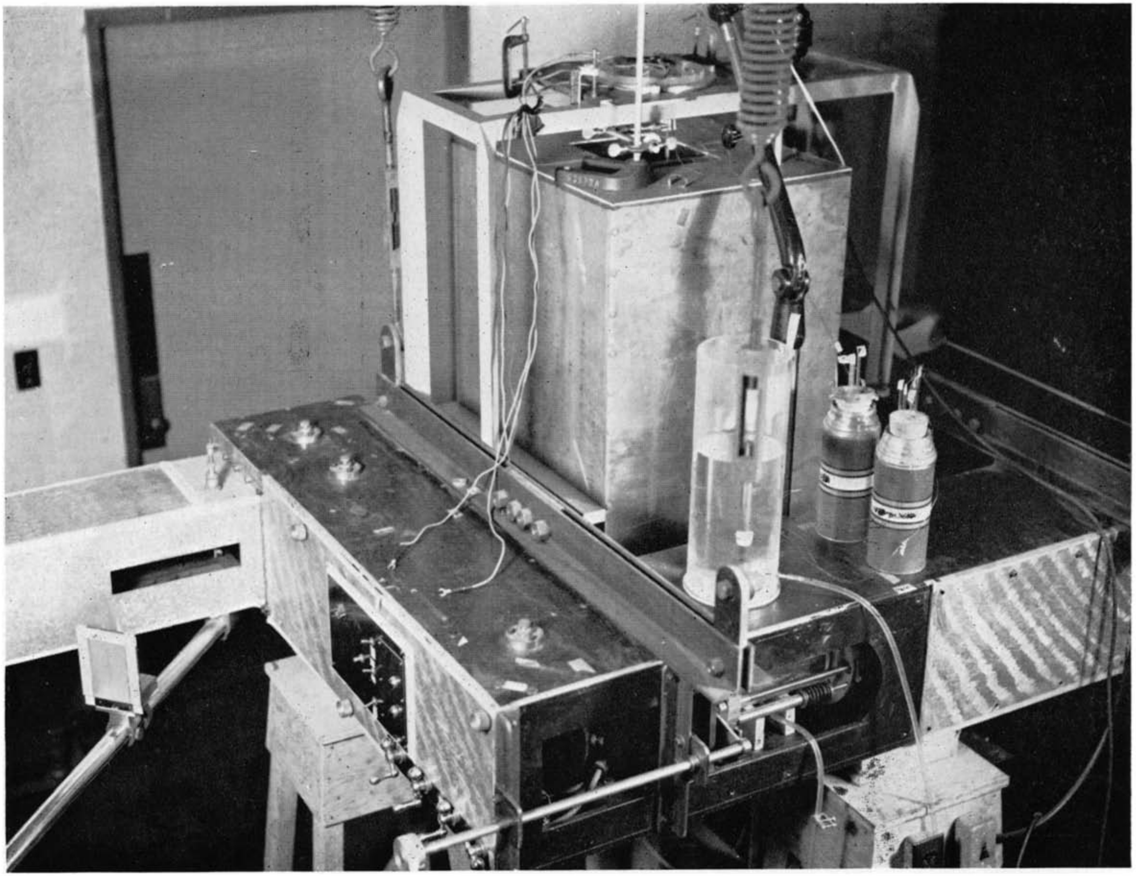

FIG. 3. Test apparatus.

effects have penetrated the entire channel length, Figs. 1(b) and 1(d). To a first approximation, the deflected fringes can be interpreted as representing the inversion of the local temperature profile in the fluid.

The previously discussed regions in the channel occurring during the early stages of the heating are clearly visible in Fig. 1(a). It can be easily noticed that the temperature profiles show a definite lack of dependence on the axial coordinate in the upper portion of the channel and a definite dependence on axial position in the lower portion. In Fig. 1(b), the boundary separating the two regions has passed through the entire height of the channel and quasi-steady state has been established. The described process is not immediately evident in Figs. 1(c) and 1(d). The existence of the two regions is demonstrated in the following discussions.

DESCRIPTION OF EXPERIMENTAL APPARATUS AND PROCEDURE

The internal heat generation was achieved by directing a 60 cycle alternating current through an electrolytic fluid. The vertical parallel-plate system investigated was open at the top and bottom. The inner surfaces of the parallel-plates were used to impress the electric potential across the fluid. Since the region immediately adjacent to the walls is of primary interest in determining the wall heat flux, electrochemical side effects at the liquid–solid interface had to be kept at a minimum. This includes such phenomena as concentration gradients, polarization, crystal formation, and gas liberation. A study of the literature in the field of electrochemistry, e.g. [11] and [12], indicated that the system of two solid silver electrodes immersed in a dilute aqueous solution of silver nitrate was the best electrolytic system for this investigation.

The vertical walls of the parallel-plate channel were constructed from solid silver blanks. The dimensions of the silver plates were 5·967 in high, 3·986 in wide, and 0·487 in thick. A balance of optical errors, which arise due to the integrating nature of the light beam of the interferometer, determined the width of the channel. The silver plates were made quite thick in order to minimize spatial and timewise variations in the wall temperature since they were not heated or cooled by external means. Two Plexiglas strips fastened to the side edges of the plates near the top of the channel controlled the spacing to within 0·001 in. All surfaces of the channel, except the inner faces, were electrically insulated with Teflon tape. Two thin pieces of optical quality glass covered the sides of the channel perpendicular to the light beam. Initial experiments indicated that the two-dimensional character of the flow would be severely disturbed if the sides of the channel were left open. A sketch of the channel with nomenclature is given in Fig. 2.

The plates were suspended in a liquid tank by means of four $\frac{1}{8}$ in copper rods, two per plate, which served simultaneously as voltage leads. The copper rods were in turn connected to an adjustable holder which was fastened to a bridge mounted on the interferometer. A photograph of the experimental apparatus showing the holder in place is given in Fig. 3.

The liquid tank, constructed from $\frac{1}{2}$ in thick aluminum plates, was 24 in in the light beam direction, 20 in width, and 36 in in height. Two optically ground glass plates 6 in in diameter and 1 in thick, which were necessary for the passage of the light beam through the tank, were in stalled by means of two aluminum tubes attached to opposite sides of the tank. The tubes

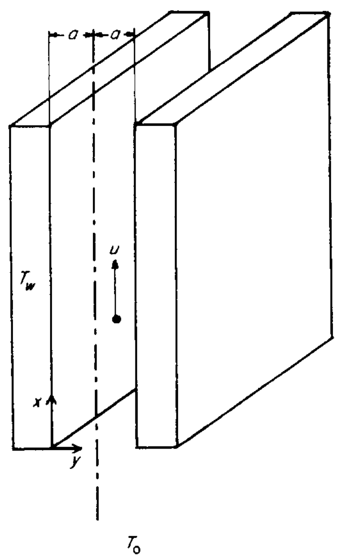

FIG. 2. Nomenclature for the parallel-plate channel.

and the tank gave a width of fluid of 8 in in the light beam direction and a field of view of about 5¼ in high. The tank allowed a maximum height of 25 in of liquid above the exit of the channel. The inner surfaces of the tank, except for the glass plates, were covered with a layer of household paraffin to eliminate possible contamination of the silver-nitrate solution.

The concentration of the aqueous solution was 3·398 g of silver nitrate per 1000 g of water. In order to prevent the formation of insoluble silver chloride, triple distilled water with a specific resistance greater than $10^6\ \Omega$ was used.

A 220/25 V transformer, which received its primary voltage from a standard 110 V supply through a step-up Variac, supplied the electrical power. The current flow was measured with a Westinghouse Type PY 5 ammeter which had a calibrated accuracy of ¼ per cent full scale. For the lower currents, the voltage drop across an accurately known resistance determined the current flow. The voltage measurements were made with a Ballantine Model 320 voltmeter which had a calibrated accuracy of 1 per cent at any point on the various scales. Since alternating current was used and the possibility of capacitance effects at the solid–liquid interface existed, the power was also measured in some instances with a modified Weston Model 432 wattmeter. The wattmeter gave, within its calibrated accuracy of ½ per cent full scale, no indication that the voltage was lagging the current.

The temperature of the solution external to the channel was measured with a bank of five 36-gage copper–constantan thermocouples connected in series. The thermocouples were placed in a kerosene-filled glass tube located near the entrance of the channel. The electromotive force of the thermocouple bank was measured with a Brown–Rubicon self-balancing potentiometer. A calibration of the thermocouples against an N.B.S. calibrated resistance thermometer indicated that the temperature of the solution could be determined to better than 0·02 degC.

The liquid tank with the parallel-plate channel was inserted in one of the two light beams of a Zehnder–Mach interferometer. In order to equalize the differences in the optical path length of the two beams, a variable width compensating chamber containing pure water was placed in the other beam. A detailed description of an interferometer similar in design to the one used here and the procedures used for its alignment are given in [13]. The method used to align the parallel-plate channel in the light beam is discussed in detail in [14]. For all runs, the interferometer is adjusted such that horizontal fringes appear in the interferogram when the field of view has a locally uniform density. Local heating of the fluid causes a downward shift of the fringes as shown in Fig. 1.

The maximum temperature difference in the experiments was approximately 1·5 degF. Extreme care had to be exercised, therefore, to insure that the temperature of the solution did not vary appreciably during the duration of a run. When the bulk temperature of the solution varied by more than 0·1 of a fringe (0·01 degF) during a run, the results were discarded. Before the start of each run a stirrer was used to decrease any temperature differences that might exist within the liquid in the test tank. After stirring and before starting the heat generation the tank was allowed to sit for a 30-min period to eliminate any possibility of motion in the liquid. All interferograms, including the initial fringe pattern, were recorded on Plus-X reversal movie film with a 16 mm camera driven by a constant speed motor. As shown in Fig. 1, a timer was simultaneously recorded on the movie film. The power, which was first set to a prescribed value, was activated by a hand-operated knife switch. During the run the voltage drop, the current flow, and, in some cases, the direct power dissipation in the channel were recorded.

REDUCTION OF EXPERIMENTAL DATA

The voltage–current measurements for four channel spacings are shown in Fig. 4. The voltage varies linearly with current for all spacings; however, the extrapolation to zero current shows a slight overvoltage. A similar overvoltage has been previously observed in electrolytic analogs; for example, by Langmuir *et al.* [15]. It is attributed to electro-chemical side effects at the liquid–solid interface, and for the results given in the next section, it amounts to at most 2·5 per cent of the voltage drop across

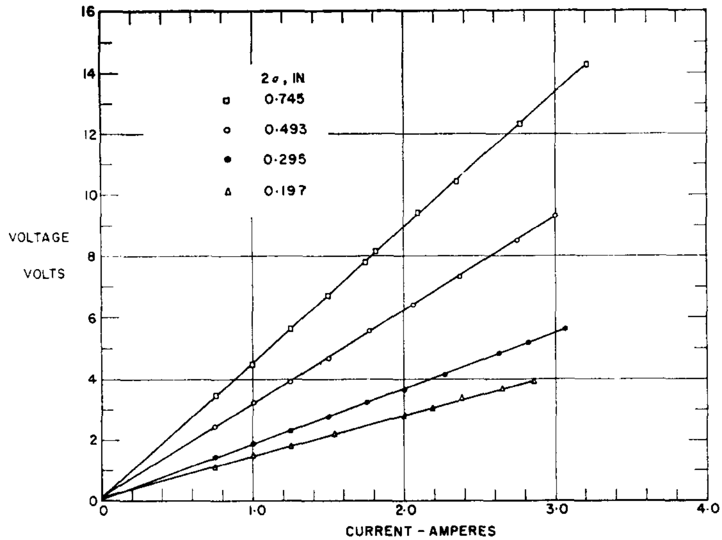

FIG. 4. Current–voltage characteristic of the aqueous solution of silver nitrate.

the electrolytic cell. Since any dissipation of energy due to the overvoltage is concentrated at the plate surface, the voltage drop across the cell was corrected for this overvoltage. The current in Fig. 4 includes the stray currents at the entrance and exit of the channel. The local current density in the space between the plates was calculated from the specific resistance of the electrolyte with values of the equivalent conductance of silver-nitrate solutions given in [16], [17] and [18]. The heat generation rates reported in this paper are based on the corrected voltage and the calculated current density.

The evaluation of the interference photographs is described in detail in [14] and is not repeated here. Two sources of systematic error have to be considered in evaluating the accuracy of the temperature distributions obtained from interference photographs. The first error occurs due to refraction effects. The second error, which in part compensates for the refraction error in this study, arises due to the fact that the interferometer integrates the density along the light beam including the region of the channel near the glass walls. A complete description of the error calculations for this study is given in [14].

The fluid properties used in the evaluation of the interferograms and in the presentation of the final results were those of pure water at the entrance temperature T_0. Results of various investigations for the density, specific heat, dynamic viscosity, and thermal conductivity of dilute silver-nitrate solutions near room temperature given in [16] and [19] clearly show the solution properties differ from those of water by no more than $\frac{1}{2}$ per cent. A calculation carried out in [14] shows that using values of the index of refraction for water [20] in the evaluation of the interference photographs introduces an error of less than 2 per cent. The effect of this error on the presented quasi-steady results is further diminished by presenting the heat-transfer data in the form of a ratio of a temperature gradient to a temperature difference.

RESULTS

Figures 5 to 15 present the results of the experiments for plate spacings of 0·197 and 0·745 in. The spacings correspond to the ones shown in Fig. 1; however, the heat generation rate for each spacing is about double. The interferograms were selected to present clear reproductions.

Figures 5 and 6 show representative temperature profiles for the transient period. Profiles for other times, heat generation rates, and spacings are presented in [14]. The temperature distribution for the largest axial distance in each

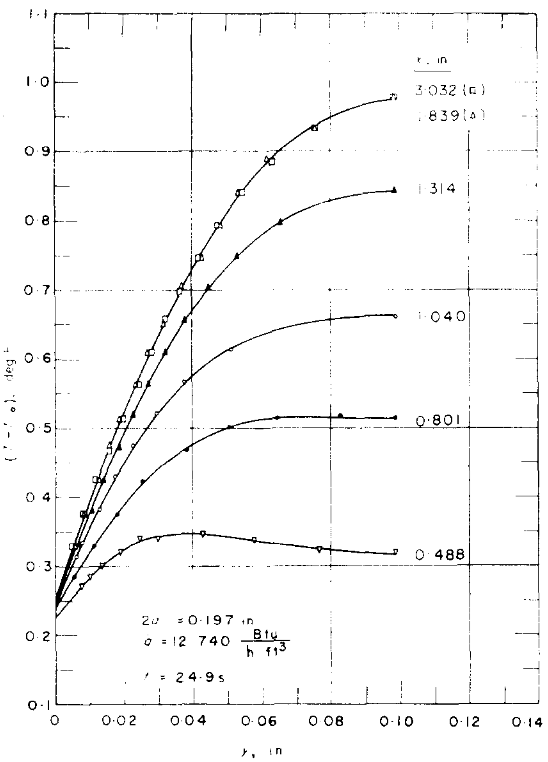

FIG. 5. Temperature profiles in the parallel-plate channel with 0·197 in spacing for unsteady conditions.

figure represents the transient one-dimensional heat-conduction region of the channel. This is indicated by the fact that two profiles at different axial positions overlap. The profiles at smaller axial distances represent the intermediate state of the fluid between the transient conduction and quasi-steady state convection. The axial position of the surface separating the conduction and convection regions of the channel is not clearly defined in the figures; a more complete discussion of this follows. The slight overshoot in temperature shown in Fig. 6 for $x = 2{\cdot}0$ in is caused by a hot spot which develops near the wall at the channel entrance. This overshoot, which propagates downstream with the flow, is caused by current fringing and is more pronounced in the beginning stages of heating when the flow rate is small. In the one-dimensional conduction region of the channel, the wall temperature, as can be observed in Figs. 5 and 6, is independent of x and very closely approximates a linear timewise variation.

Figures 7 and 8 represent the temperature distributions in the channel for the wide and narrow spacing, respectively, after the convection processes have become quasi-steady. It can be observed that the wall temperature exhibits a definite dependence on axial position. If the wall temperatures in Figs. 7 and 8 are plotted as a function of axial position, it is found that the relationship is approximately linear. Except near the entrance of the channel, the deviation from a linear variation is less than 0·02 degF. Thus, the wall temperature is spatially uniform during the early stages of heating and changes to a linear variation with axial position at the quasi-steady state. This complicates the physical processes during the interim period of time and makes a theoretical analysis for this period difficult.

Figures 9 and 10 present the centerline temperatures for the narrow and wide spacings, respectively, plotted as a function of time from the start of the heat generation. The experimental results in the conduction region of the channel (represented by the open circles in Fig. 9) are compared to a theoretical solution taken from [21] for transient one-dimensional conduction with uniform internal heat generation where the wall boundary condition is given by

$$T_w - T_0 = A\,t \qquad (1)$$

The constant A used for the analysis in Figs. 9 and 10 was determined by a linear approximation to the experimental wall temperature variation. The agreement between theory and experiment is very good.

The times at which the centerline temperature starts to depart from the one-dimensional conduction solution are shown in Figs. 9 and 10 as solid vertical lines. These departure times were determined from a cross-plot of the centerline temperature versus axial position as shown in Fig. 11 for the narrow spacing. The intersection of the two dashed lines determines, to a first approximation, the location of the surface separating the two regions on the centerline. The axial positions obtained in this manner were then plotted versus t to obtain the results given in Figs. 9 and 10. The value given for $x = 6{\cdot}0$ in

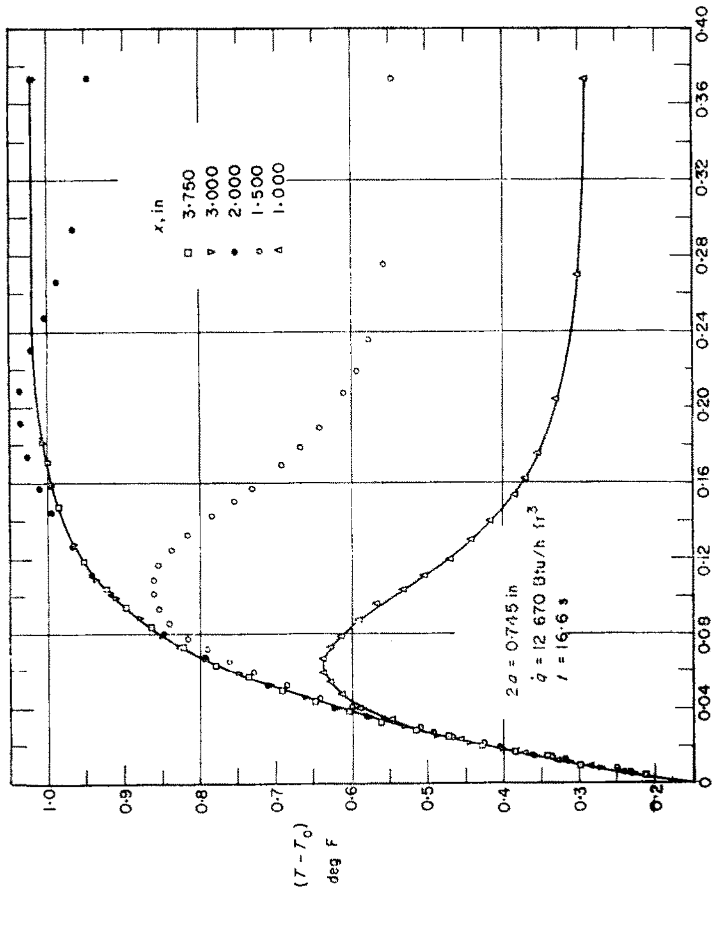

FIG. 6. Temperature profiles in the parallel-plate channel with 0·745 in spacing for unsteady conditions.

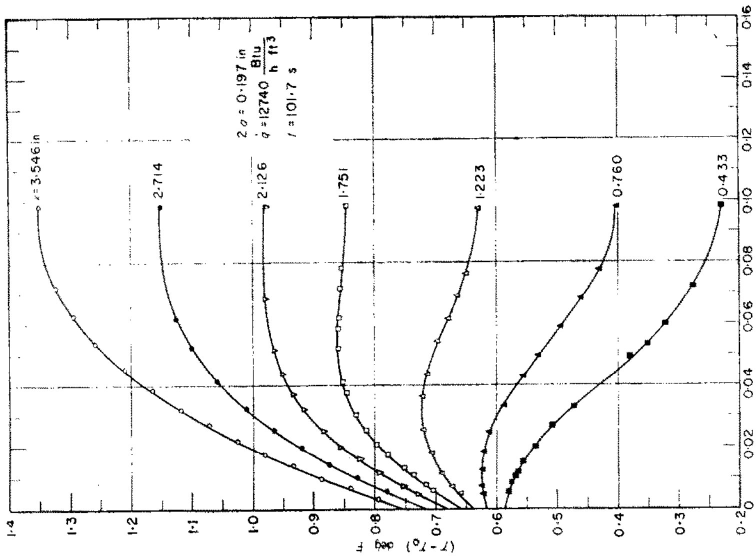

FIG. 7. Temperature profiles in the parallel-plate channel with 0·197 in spacing for quasi-steady conditions.

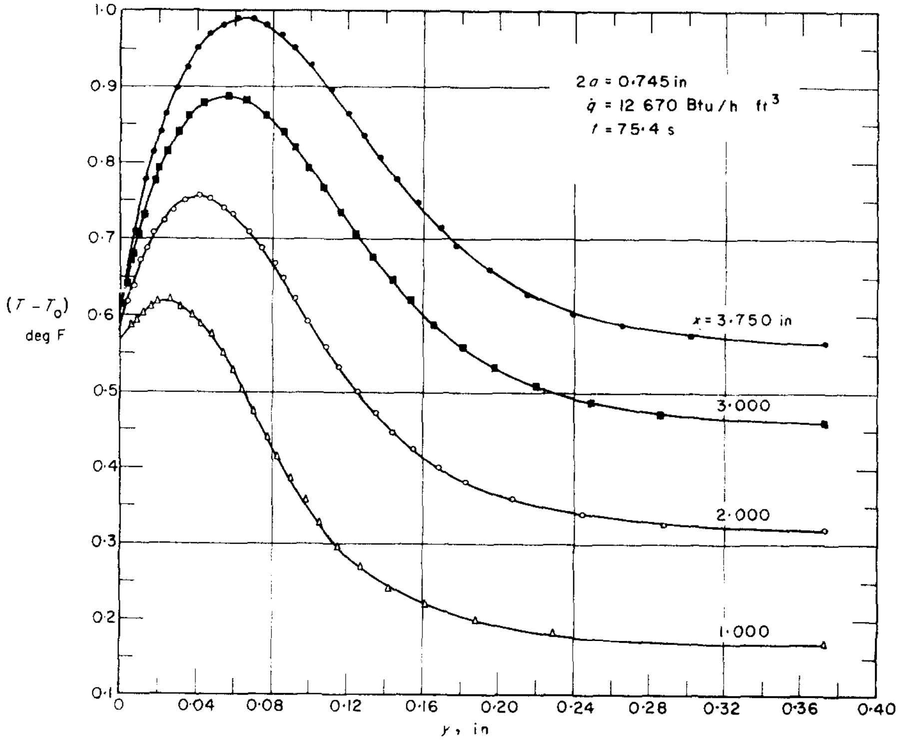

FIG. 8. Temperature profiles in the parallel-plate channel with 0·745 in spacing for quasi-steady conditions.

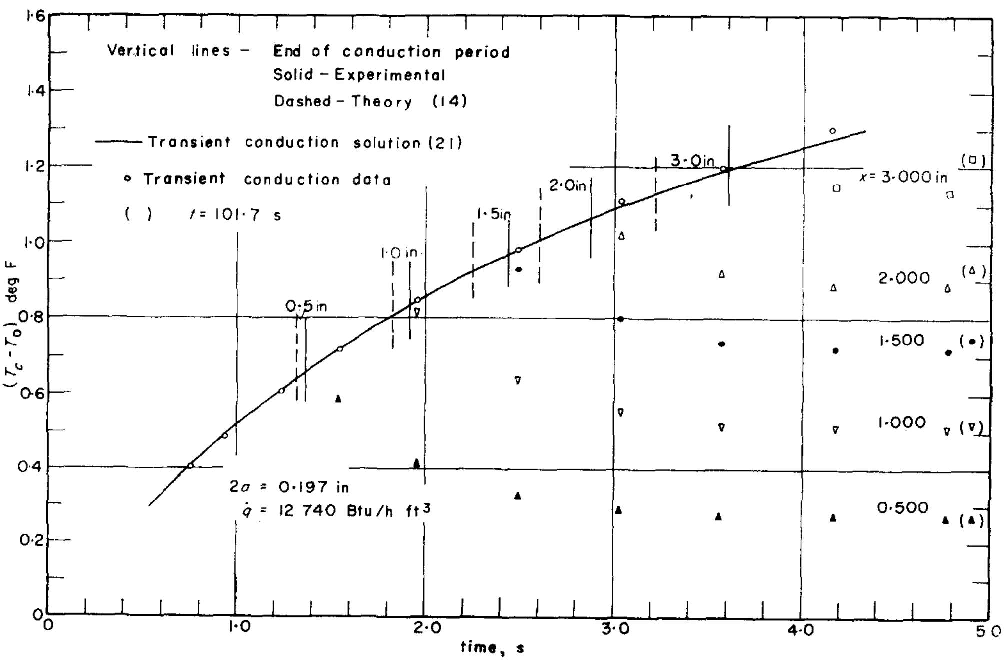

FIG. 9. Variation of the centerline temperature of the fluid with time for the parallel-plate channel with 0·197 in spacing.

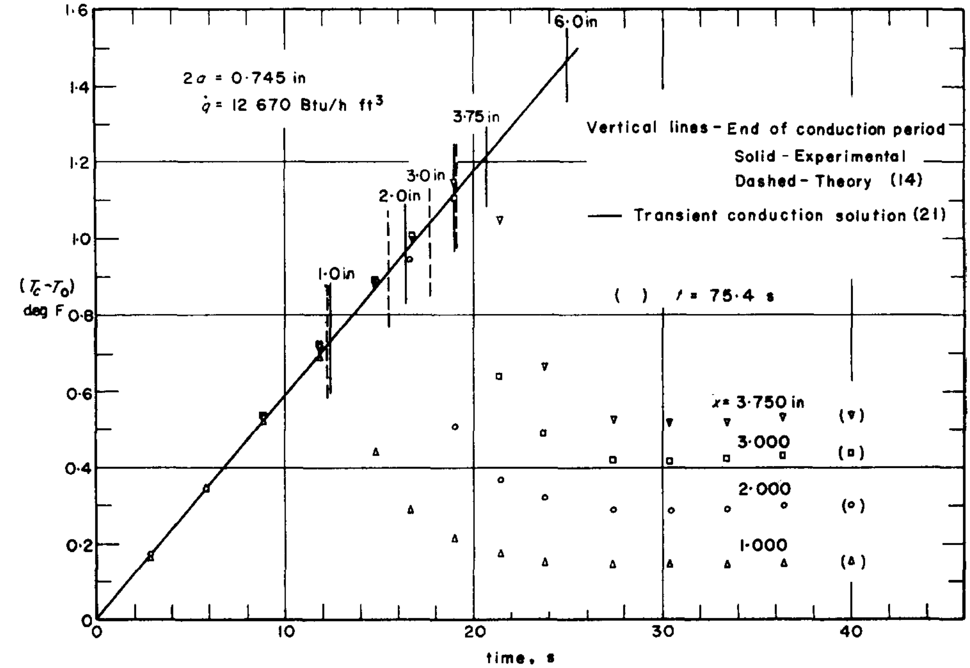

FIG. 10. Variation of the centerline temperature of the fluid with time for the parallel-plate channel with 0·745 in spacing.

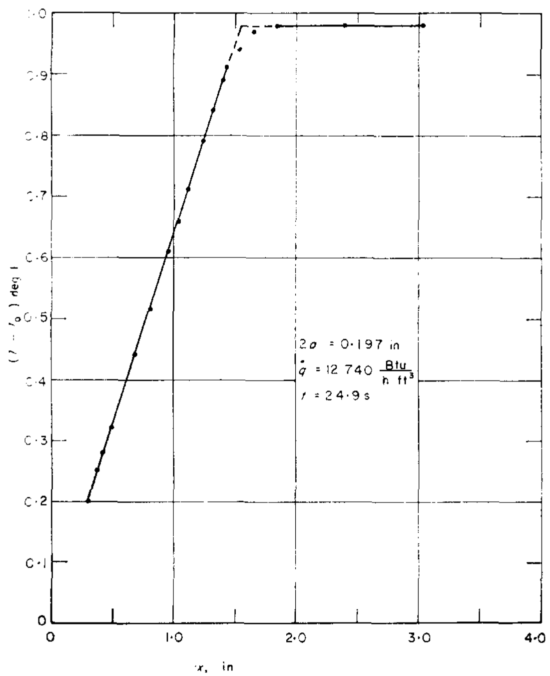

FIG. 11. Variation of the centerline temperature of the fluid with axial coordinate for the time of 24·9 s in the parallel-plate channel with 0·197 in spacing.

was obtained by extrapolation. Shown as dashed lines in Figs. 9 and 10 are departure times obtained by an analysis discussed in reference 14. In the analysis, the departure time is taken as the time which a fluid particle located on the centerline of the channel entrance at the beginning of heating needs to reach a prescribed position x. The velocity of the particle is determined from a solution of the momentum equation for a doubly infinite parallel-plate channel with the buoyancy force prescribed by the temperature distribution for one-dimensional unsteady conduction. Figures 9 and 10 exhibit a reasonable agreement between the calculated and measured departure times as long as the distance x is not too large.

The dimensionless heat flux, $q/\dot{q}a$, from the fluid into the duct walls is presented as a function of time for the two spacings in Figs. 12 and 13. The experimental data are compared with the results of an analysis for the early transient conduction period, presented as a solid line, and for the quasi-steady state, presented as dashed lines. As in Fig. 9, the open circles in

Fig. 12 represent data for various values of x in the transient conduction region of the channel.

The analysis for the conduction period is made by approximating the actual conditions by a one-dimensional unsteady conduction process as was previously discussed. The vertical lines intersecting the conduction curves are taken from Figs. 9 and and 10 and indicate the times at which the centerline temperature departs from the time-increase characteristic of the conduction period. For the narrow spacing, Fig. 12, the heat flux departs from the values predicted by the conduction analysis at a time quite close to that predicted by the experimental centerline temperature. For the wide spacing, Fig. 13, the departure of the wall flux from the theory lags quite noticeably behind that of the centerline temperature. Such a delay is understandable since the convection wave which started from the entrance at the beginning of the heating moves slower in the regions near the wall.

For the analysis of the heat transfer under quasi-steady conditions, the results of which are presented as dashed curves in Figs. 12 and 13, the average velocity of the fluid in the duct must be known. Since velocities were not measured, information was obtained from the interferograms in the following way: at a location where the conditions

$$\frac{\partial T}{\partial y} = 0, \quad \frac{\partial^2 T}{\partial y^2} = 0, \quad \text{and} \quad \frac{\partial^2 T}{\partial x^2} = 0$$

are satisfied, the energy equation describing the quasi-steady temperature field in a heat generating the quasi-steady temperature field in a heat generating fluid reduces to

$$u \frac{\mathrm{d}T}{\mathrm{d}x} = \frac{\dot{q}}{\rho c} \tag{2}$$

The first condition is fulfilled along the centerline of the duct. The second condition is satisfied in the narrow duct where the curvature of the temperature profile changes from a positive to a negative value. For the wide spacing, the second condition is satisfied along the centerline near the channel entrance. These conclusions can be easily verified in Figs. 1(b) and

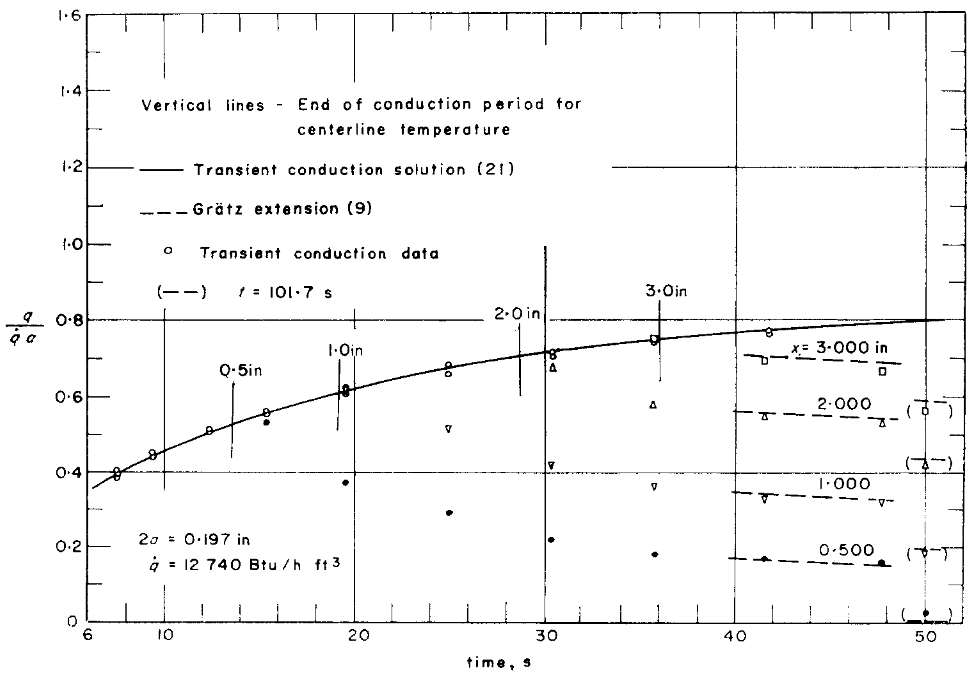

FIG. 12. Heat flux to the wall of the parallel-plate channel with 0·197 in spacing as a function of time.

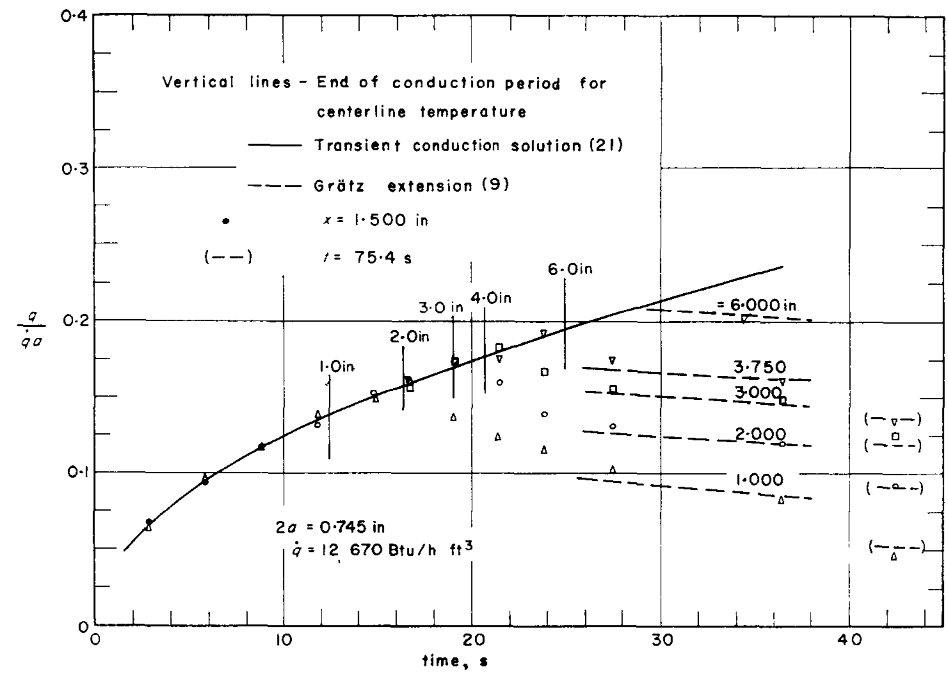

FIG. 13. Heat flux to the wall of the parallel-plate channel with 0·745 in spacing as a function of time.

1(d). An examination of the centerline temperature data indicated that the last condition was also a reasonable approximation.

The average velocity in the narrow spacing was then calculated from the centerline velocity with the equation

$$\bar{u} = \frac{2}{3} u_c \qquad (3)$$

assuming that the velocity profile can be approximated by a parabola. The degree of approximation introduced by equation (3) can be estimated from an analysis for a fully developed velocity and temperature field created by buoyancy forces in a parallel-plate channel. This analysis is given in [14] and follows closely the one presented in [1] for a circular tube. The resulting velocity profile indicates that local buoyancy should have less than a 1 per cent effect on the average velocity calculated by equation (3). Additionally, the hydrodynamic entrance length, as estimated from the results given in [22], is quite small. Thus, disregarding end effects, the use of a parabolic velocity profile to determine the average velocity in the narrow duct is justified. The average velocity in the wide spacing was calculated from the centerline velocity by use of a relation which arises from solution of the integrated momentum equation for laminar forced flow in the entrance region of a parallel-plate duct [14, 22]. A complete description of these calculations is given in [14]. The average velocity obtained in the above manner for both spacings is estimated to be in error by 5 per cent at most.

The prediction of the heat flux q given in Figs. 12 and 13 for the quasi-steady region is the result of an extension of the Grätz analysis [9] to include internal heat sources in the fluid and a spatial wall temperature variation of the form

$$T_w - T_0 = Bx + (T_w - T_0)_0 \qquad (4)$$

The constants B and $(T_w - T_0)_0$ were obtained from a linear approximation to the experimental wall temperature variation. In the wide spacing, the contribution of the term Bx was small compared to the term $(T_w - T_0)_0$. Thus, the wall temperature was assumed to be constant

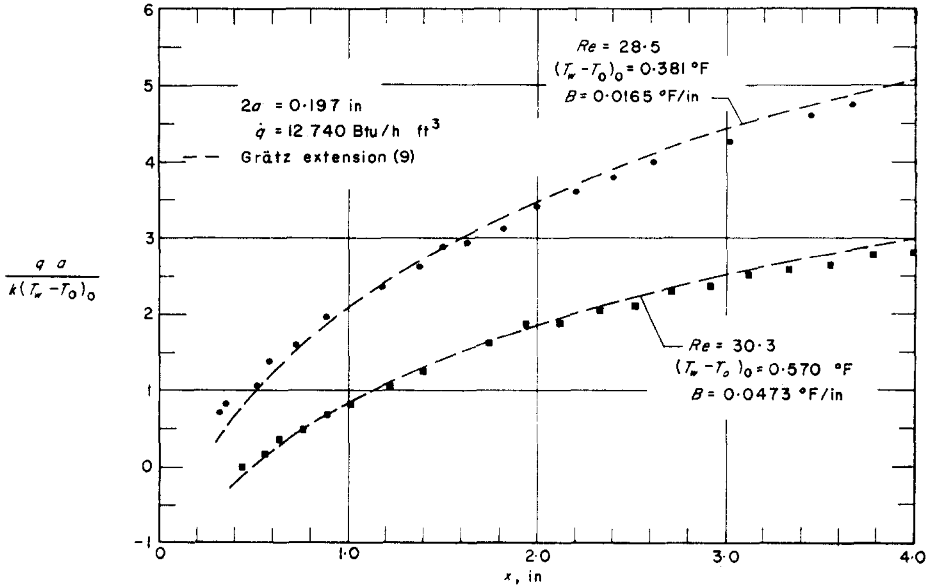

FIG. 14. Heat flux to the wall of the parallel-plate channel with 0·197 in spacing for quasi-steady conditions as a function of axial co-ordinate.

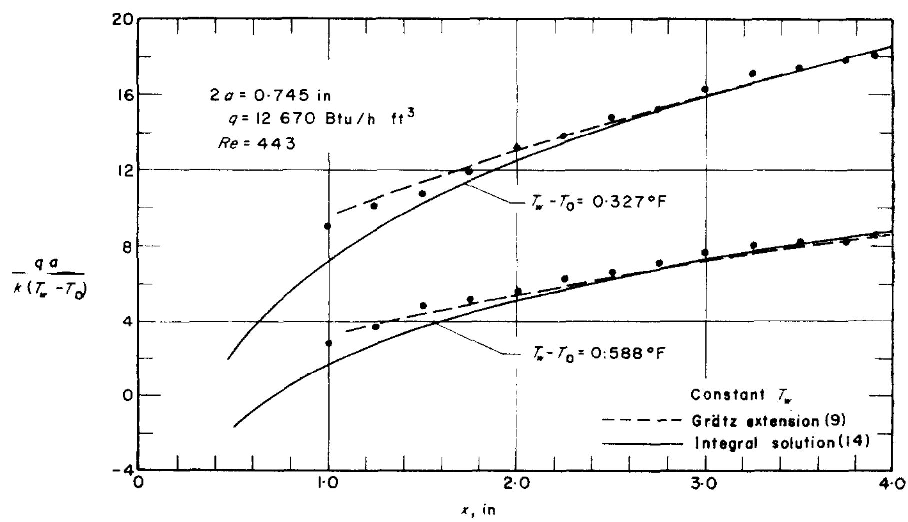

FIG. 15. Heat flux to the wall of the parallel-plate channel with 0·745 in spacing for quasi-steady conditions as a function of axial co-ordinate.

for purposes of comparison of theory to experiment. The comparison of the results of the Grätz extension with the experimental points indicates satisfactory agreement even for the flow development region in the wide spacing. This was also found to be the case for two additional plate spacings and for a smaller heat generation rate [14].

Figures 14 and 15 present the heat flux q for the last two interferograms (the last two values of time in Figs. 12 and 13) in the form of a different dimensionless parameter as a function of the axial distance x. The dashed lines in the two figures are again the results of the Grätz extension [9]. The solid lines in Fig. 15 are based on a boundary-layer solution for forced laminar flow using the integrated momentum, energy, and ontinuity equations. This analysis, which takes into account the simultaneous development of the flow and temperature fields, is described in reference 14. As shown in the figures, the experimental results agree quite well with the Grätz extension; however, an error analysis in [14] indicates that the experimental points are slightly high for small values of x.

In summary, it is felt that for systems similar to the one investigated here, the local heat transfer can be predicted with engineering accuracy for the whole period of time from the start of the heating to steady or quasi-steady conditions. The initial period is described with good approximation by the one-dimensional unsteady heat conduction equation. The end of this conduction period can be approximated by the time which a fluid particle needs to move along the centerline of the duct from the entrance to the specified cross section. The steady or quasi-steady heat transfer can be estimated from a forced convection analysis provided the flow parameters are known. If the overall flow is caused solely by free convection, as is the case here, a first approximation to the average velocity can be obtained from a simple force balance on the plate system. The results of such a calculation [14] for the narrow spacing agree with the average velocity obtained from the interferograms to within 6 per cent. For the region between the initial conduction period and quasi-steady state, the heat flux can be obtained with reasonable accuracy by interpolation.

REFERENCES

1. T. M. HALLMAN, Combined forced and free convection in a vertical tube. Ph.D. Thesis, Purdue University, May (1958).
2. S. OSTRACH, Combined natural-convection flow and heat transfer of fluids with and without heat sources in channels with constant wall temperatures. *NACA TN* 2863 (1952).
3. S. OSTRACH, Combined natural- and forced-convection laminar flow and heat transfer of fluids with and without heat sources in channels with linearly varying wall temperatures. *NACA TN* 3141 (1954).
4. H. F. POPPENDICK and L. D. PALMER, Forced-convection heat transfer in pipes with volume heat sources within the fluids. *ORNL* 1395 (1952).
5. H. F. POPPENDICK, Forced-convection heat transfer in pipes with volume sources within the fluids. *Nuclear Engng, Part I: Chemical Engng Progress Symposium Series*, No. 11, **50**, 93–104 (1954).
6. H. F. POPPENDICK and L. D. PALMER, Forced convection heat transfer between parallel plates and in annuli with volume heat sources within the fluids. *ORNL* 1701 (1954).
7. L. TOPPER, Forced heat convection i· cylindrica channels, *Chem. Engng Sci.* **5**, 13–19 (1956).
8. E. M. SPARROW and R. SIEGEL, Laminar tube flow with arbitrary internal heat sources and wall heat transfer, *Nucl. Sci. Engng* **4**, 239–254 (1958).
9. E. M. SPARROW, J. L. NOVOTNY and S. LIN, Flow of a heat-generating fluid in a parallel-plate channel, *J. Amer. Inst. Chem. Engrs* **9**, 797–804 (1963).
10. R. M. INMAN, Experimental study of temperature distribution in laminar tube flow of a fluid with internal heat generation, *Int. J. Heat Mass Transfer* **5**, 1053–1058 (1962).
11. F. H. NEWMAN, *Electrolytic Conduction*. Chapman-Hall, London (1930).
12. S. GLASSTONE, *The Electrochemistry of Solutions*. Van Nostrand, New York (1930).
13. E. R. G. ECKERT, R. M. DRAKE and E. SOEHNGEN, Manufacture of a Zehnder–Mach interferometer. Air Force Report 5721, *ASTIA* Document No. 34235 (1948).
14. J. L. NOVOTNY, Interferometric study of laminar convection in a vertical parallel plate channel with uniform heat sources in the fluid. Ph.D. Thesis, University of Minnesota (1963).
15. I. LANGMUIR, E. Q. ADAMS and G. S. MEIKLE, Flow of heat through furnace walls: The shape factor, *Trans. Electrochem. Soc.* **24**, 53–84 (1913).
16. *International Critical Tables*. McGraw-Hill, New York (1930).
17. A. N. CAMPBELL and R. J. FRIESEN, Conductance in the range of medium concentration. *Canad. J. Chem.* **37**, 1288–1293 (1959).
18. T. SHEDLOVSKY, The electrolytic conductivity of uni-valent electrolytes in water at 25°C, *J. Amer. Chem. Soc.* **54**, 1411–1428 (1932).
19. B. SAKIADIS and J. COATES, Literature survey of the thermal conductivity of liquids. Louisiana State

University Experiment Station Bulletin No. 34 (1952).
20. L. TILTON and J. J. TAYLOR, Refractive index and dispersion of distilled water for visible radiation at temperatures 0 to 60°C, *J. Res. Nat. Bur. Stand., Wash.* **20**, 419 (1938).
21. H. CARSLAW and J. JAEGER, *Conduction of Heat in Solids*, Second edition. Oxford University Press (1959).
22. E. M. SPARROW, Analysis of laminar forced convection heat transfer in entrance region of flat rectangular ducts. *NACA TN* 3331 (1958).

Résumé—Les résultats du transport de chaleur pour l'écoulement par convection naturelle d'un fluide avec sources internes de chaleur dans un canal vertical à plaques parallèles ont été obtenus à l'aide d'un interféromètre de Zehnder–Mach. Les expériences furent conduites avec un canal à plaques parallèles, ouvert au sommet et dans le bas, suspendu dans un grand réservoir d'électrolyte dilué. Les caractéristiques de transport de chaleur du canal furent étudiées en partant d'un état initial de température uniforme dans tout le système (pas d'écoulement) jusqu'au début de la convection quasi-permanente lorsqu'un changement brusque dans la génération de chaleur est appliqué au fluide initialement entre les parois du canal.

Les résultats expérimentaux sont présentés pour des espacements du canal de 5 et 18,9 mm et pour un débit de chaleur engendrée d'approximativement $3{,}75.10^{-2}$ watts/cm^3. Ces résultats sont comparés avec les analyses théoriques pour le début de la période transitoire et pour l'état final quasi-permanent. L'accord entre l'éxpérience et la théorie indique que le transport de chaleur pour la gamme de paramètres étudiée peut être prédite convenablement pour toute la période transitoire.

Zusammenfassung—Wärmeübergangsergebnisse für freie Konvektionsströmung in einer Flüssigkeit mit Wärmequellen liessen sich mit Hilfe eines Zehnder–Mach-Interferometers an einem senkrechten, parallelwandigen Kanal erhalten. Dieser Kanal war oben und unten offen und hing in einem grossen Behälter mit verdünntem Elektrolyt. Charakteristiken des Wärmeübergangs im Kanal wurden untersucht, wobei man von einem Anfangszustand mit gleichmässiger Temperatur im ganzen System (keine Strömung) ausging und das Einsetzen der quasi-stationären Konvektion, bei einer stufenweisen Wärmeerzeugung in der ursprünglich zwischen den Kanalwänden ruhenden Flüssigkeit beobachtete.

Versuchsergebnisse liegen vor für Kanalweiten von 5 mm und 19 mm und einer Volumenleistung der inneren Wärmequellen von etwa 134 kw/m^3. Diese Ergebnisse werden mit theoretischen Betrachtungen für den Beginn der Anlaufperiode und für den endgültigen quasi-stationären Zustand verglichen. Die Übereinstimmung zwischen Versuch und Theorie zeigt, dass der Wärmeübergang im Bereich der untersuchten Parameter für die ganze Anlaufperiode gut berechnet werden kann.

Аннотация—Приводятся результаты исследования теплообмена при свободной конвекции тепло-генерирующей жидкости в вертикальном канале между параллельными пластинами, полученные с помощью интерферометра Цендера-Маха. Опыты проводились в канале, открытом сверху и снизу, образуемом паралельными пластинами, помещенными в большом резервуаре с разбавленным электролитом. Исследовались теплообменные характеристики канала, начиная с равномерной температуры в начальном состоянии всей системы (течение отсутствует) до начала квазистационарной конвекции, когда ступенчатое изменение генерации тепла прикладывается к жидкости между стенками канала.

Представлены экспериментальные результаты для канала шириной 0,197 и 0,745 дюймов для интенсивности внутренней генерации тепла приблизительно 13000 бет. Эти результаты сравниваются с теоретическим анализом начальной стадии переходного процесса и конечного квазистационарного состояния. Соответствие между экспериментом и теорией показывает, что теплообмен для исследованных параметров можно рассчитать для нестационарного процесса в целом.